Graphs of Polynomial Functions
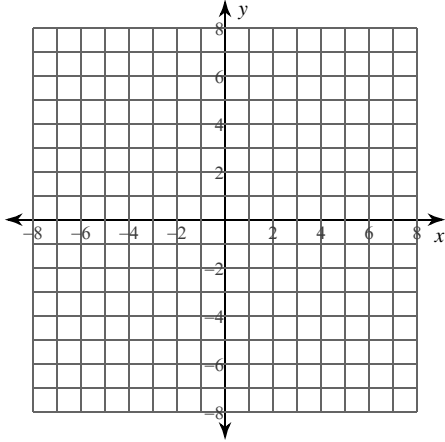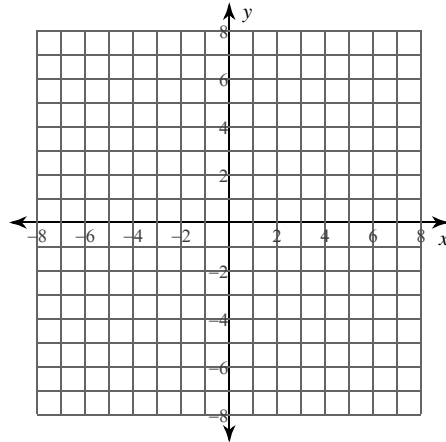
For each function: (1) determine the real zeros and state the multiplicity of any repeated zeros, (2) list the x-intercepts where the graph crosses the x-axis and those where it does not cross the x-axis, and (3) sketch the graph.
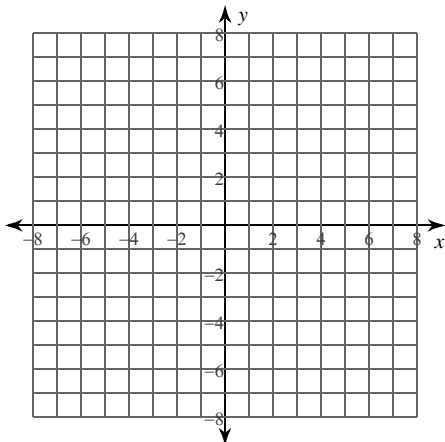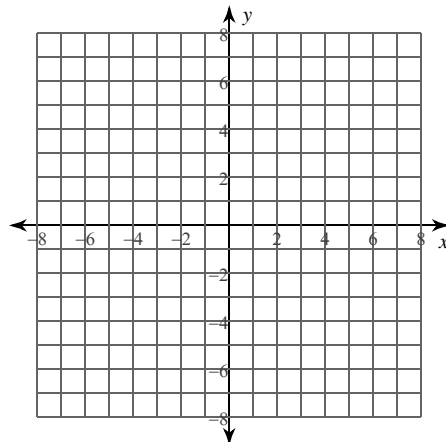
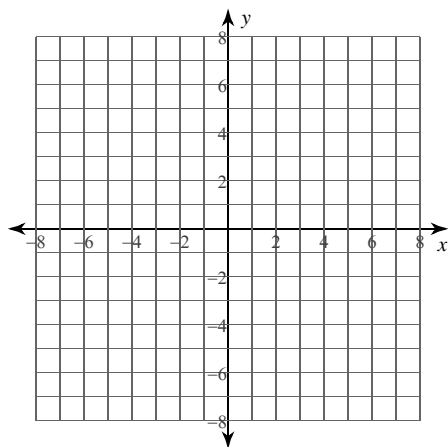1) $f(x) = -x^3$
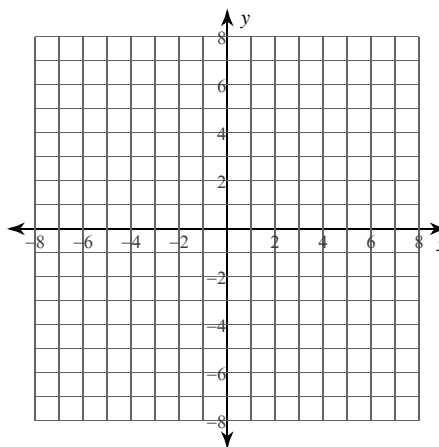
2) $f(x) = 2x^3 - 3x^2$

3) $f(x) = x^4 + x^3 - 4x^2 - 4x$

4) $f(x) = x^4 + x^3$

5) $f(x) = -x^3 + 6x^2 - 12x + 8$

6) $f(x) = x^3 - 2x^2$

Describe the end behavior of each function.

7) $f(x) = -x^5 + 2x^3 - x + 1$

8) $f(x) = 2x^2 - 4x - 3$

9) $f(x) = x^4 - 2x^2 - x + 1$

10) $f(x) = -x^3 - 9x^2 - 24x - 20$

11) $f(x) = -x^5 + 3x^3 + 1$

12) $f(x) = x^2 + 6x + 6$

Critical thinking questions:

13) Write a polynomial function f with the following properties:
   (a) Zeros at 1, 2, and 3
   (b) $f(x) \leq 0$ for all values of x
   (c) Degree greater than 1

14) Write a polynomial function g with degree greater than one that passes through the points $(0, 1)$, $(1, 1)$, and $(2, 1)$.
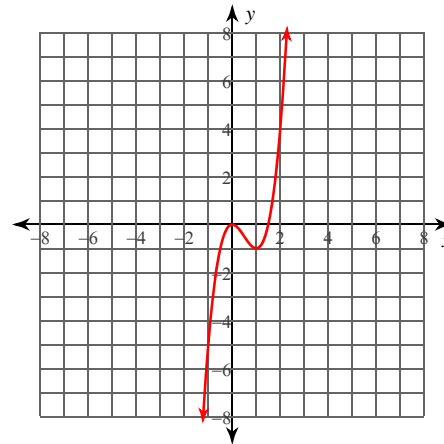
Graphs of Polynomial Functions

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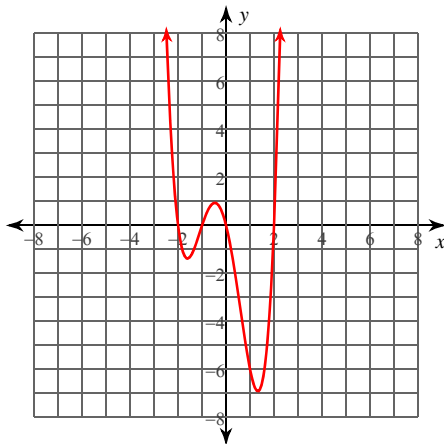
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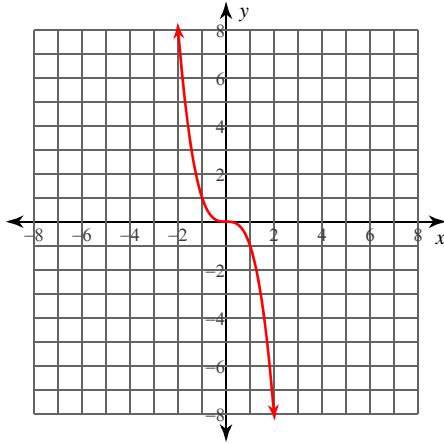
Real zeros: $\{0 \text{ mult. } 3\}$
x-int, crosses: 0
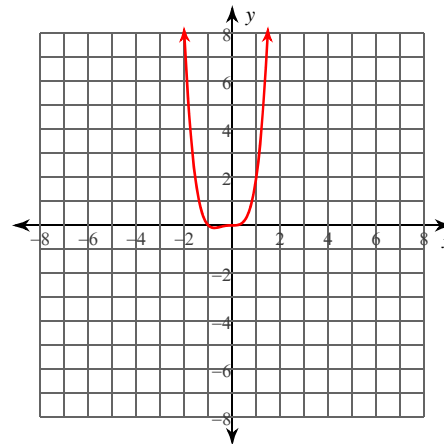x-int, doesn't cross: None

2) $f(x) = 2x^3 - 3x^2$

Real zeros: $\left\{0 \text{ mult. } 2, \frac{3}{2}\right\}$

x-int, crosses: $\frac{3}{2}$

x-int, doesn't cross: 0
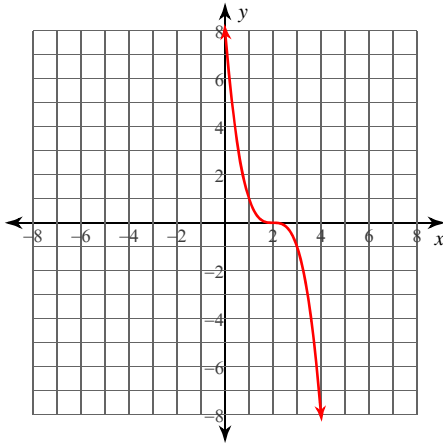
3) $f(x) = x^4 + x^3 - 4x^2 - 4x$

Real zeros: $\{0, 2, -2, -1\}$
x-int, crosses: 0, 2, −2, −1
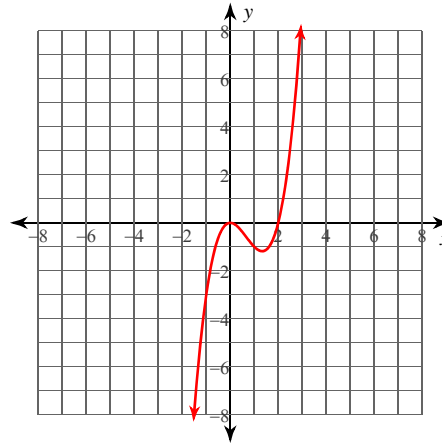x-int, doesn't cross: None

4) $f(x) = x^4 + x^3$

Real zeros: $\{0 \text{ mult. } 3, -1\}$
x-int, crosses: 0, −1
x-int, doesn't cross: None

5) $f(x) = -x^3 + 6x^2 - 12x + 8$

Real zeros: {2 mult. 3}
x-int, crosses: 2
x-int, doesn't cross: None

6) $f(x) = x^3 - 2x^2$

Real zeros: {0 mult. 2, 2}
x-int, crosses: 2
x-int, doesn't cross: 0

Describe the end behavior of each function.

7) $f(x) = -x^5 + 2x^3 - x + 1$

$$\lim_{x \to -\infty} f(x) = \infty$$
$$\lim_{x \to \infty} f(x) = -\infty$$

8) $f(x) = 2x^2 - 4x - 3$

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   (a) Zeros at 1, 2, and 3
   (b) $f(x) \le 0$ for all values of x
   (c) Degree greater than 1

   $f(x) = -(x-1)^2 \cdot (x-2)^2 \cdot (x-3)^2$

14) Write a polynomial function g with degree greater than one that passes through the points $(0, 1)$, $(1, 1)$, and $(2, 1)$.

   $g(x) = x(x-1)(x-2) + 1$

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