Candle Center - Infinite Algebra 2   Name_____

Analyzing and Solving Polynomial Equations   Date_____ Period____

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$

2) $x^3 + 3x^2 - 14x - 20 = 0$

3) $x^3 - 2x^2 + 3x - 6 = 0$

4) $x^4 - 14x^2 + 45 = 0$

5) $x^4 + 6x^2 + 8 = 0$

6) $x^4 + 3x^2 - 18 = 0$

7) $x^3 - 1 = 0$

8) $x^3 + 3x^2 - x - 3 = 0$

9) $x^3 - 2x^2 - 3x + 6 = 0$

10) $x^6 - 2x^4 - 4x^2 + 8 = 0$

11) $x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$

12) $x^6 + 5x^4 - 4x^2 - 20 = 0$

13) $x^6 - x^4 - x^2 + 1 = 0$

14) $x^8 - 26x^4 + 25 = 0$

Name_____

Analyzing and Solving Polynomial Equations

Date_____ Period_____

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$

# of complex roots: 4
Possible # of real roots: 4, 2, or 0
Possible # of imaginary roots: 4, 2, or 0
Possible # positive real roots: 1
Possible # negative real roots: 1
Possible rational roots:
   $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$
Roots: $\{2i, -2i, 3, -3\}$

2) $x^3 + 3x^2 - 14x - 20 = 0$

# of complex roots: 3
Possible # of real roots: 3 or 1
Possible # of imaginary roots: 2 or 0
Possible # positive real roots: 1
Possible # negative real roots: 2 or 0
Possible rational roots:
   $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
Roots: $\{-5, 1 + \sqrt{5}, 1 - \sqrt{5}\}$

3) $x^3 - 2x^2 + 3x - 6 = 0$

# of complex roots: 3
Possible # of real roots: 3 or 1
Possible # of imaginary roots: 2 or 0
Possible # positive real roots: 3 or 1
Possible # negative real roots: 0
Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$
Roots: $\{2, i\sqrt{3}, -i\sqrt{3}\}$

4) $x^4 - 14x^2 + 45 = 0$

# of complex roots: 4
Possible # of real roots: 4, 2, or 0
Possible # of imaginary roots: 4, 2, or 0
Possible # positive real roots: 2 or 0
Possible # negative real roots: 2 or 0
Possible rational roots:
   $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$
Roots: $\{\sqrt{5}, -\sqrt{5}, 3, -3\}$

5) $x^4 + 6x^2 + 8 = 0$

# of complex roots: 4
Possible # of real roots: 4, 2, or 0
Possible # of imaginary roots: 4, 2, or 0
Possible # positive real roots: 0
Possible # negative real roots: 0
Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8$
Roots: $\{2i, -2i, i\sqrt{2}, -i\sqrt{2}\}$

6) $x^4 + 3x^2 - 18 = 0$

# of complex roots: 4
Possible # of real roots: 4, 2, or 0
Possible # of imaginary roots: 4, 2, or 0
Possible # positive real roots: 1
Possible # negative real roots: 1
Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
Roots: $\{\sqrt{3}, -\sqrt{3}, i\sqrt{6}, -i\sqrt{6}\}$

7) $x^3 - 1 = 0$

# of complex roots: 3
Possible # of real roots: 3 or 1
Possible # of imaginary roots: 2 or 0
Possible # positive real roots: 1
Possible # negative real roots: 0
Possible rational roots: ± 1
Roots: $\left\{1, \dfrac{-1 + i\sqrt{3}}{2}, \dfrac{-1 - i\sqrt{3}}{2}\right\}$

8) $x^3 + 3x^2 - x - 3 = 0$

# of complex roots: 3
Possible # of real roots: 3 or 1
Possible # of imaginary roots: 2 or 0
Possible # positive real roots: 1
Possible # negative real roots: 2 or 0
Possible rational roots: $\pm 1, \pm 3$
Roots: $\{-3, 1, -1\}$

9) $x^3 - 2x^2 - 3x + 6 = 0$

# of complex roots: 3
Possible # of real roots: 3 or 1
Possible # of imaginary roots: 2 or 0
Possible # positive real roots: 2 or 0
Possible # negative real roots: 1
Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$
Roots: $\left\{2, \sqrt{3}, -\sqrt{3}\right\}$

10) $x^6 - 2x^4 - 4x^2 + 8 = 0$

# of complex roots: 6
Possible # of real roots: 6, 4, 2, or 0
Possible # of imaginary roots: 6, 4, 2, or 0
Possible # positive real roots: 2 or 0
Possible # negative real roots: 2 or 0
Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8$
Roots: $\left\{\sqrt{2} \text{ mult. 2}, -\sqrt{2} \text{ mult. 2}, i\sqrt{2}, -i\sqrt{2}\right\}$

11) $x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$

# of complex roots: 5
Possible # of real roots: 5, 3, or 1
Possible # of imaginary roots: 4, 2, or 0
Possible # positive real roots: 0
Possible # negative real roots: 5, 3, or 1
Possible rational roots:
  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$
Roots: $\left\{-2, i\sqrt{3}, -i\sqrt{3}, 2i\sqrt{2}, -2i\sqrt{2}\right\}$

12) $x^6 + 5x^4 - 4x^2 - 20 = 0$

# of complex roots: 6
Possible # of real roots: 6, 4, 2, or 0
Possible # of imaginary roots: 6, 4, 2, or 0
Possible # positive real roots: 1
Possible # negative real roots: 1
Possible rational roots:
  $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
Roots: $\left\{i\sqrt{5}, -i\sqrt{5}, \sqrt{2}, -\sqrt{2}, i\sqrt{2}, -i\sqrt{2}\right\}$

13) $x^6 - x^4 - x^2 + 1 = 0$

# of complex roots: 6
Possible # of real roots: 6, 4, 2, or 0
Possible # of imaginary roots: 6, 4, 2, or 0
Possible # positive real roots: 2 or 0
Possible # negative real roots: 2 or 0
Possible rational roots: ± 1
Roots: $\left\{1 \text{ mult. 2}, -1 \text{ mult. 2}, i, -i\right\}$

14) $x^8 - 26x^4 + 25 = 0$

# of complex roots: 8
Possible # of real roots: 8, 6, 4, 2, or 0
Possible # of imaginary roots: 8, 6, 4, 2, or 0
Possible # positive real roots: 2 or 0
Possible # negative real roots: 2 or 0
Possible rational roots: $\pm 1, \pm 5, \pm 25$
Roots: $\left\{1, -1, i, -i, \sqrt{5}, -\sqrt{5}, i\sqrt{5}, -i\sqrt{5}\right\}$