

Multiple-Angle Identities

Use the half-angle identities to find the exact value of each.

1) $\sin 292.5^\circ$

2) $\sin 337.5^\circ$

3) $\tan 105^\circ$

4) $\cos 105^\circ$

5) $\cos 157.5^\circ$

6) $\cos 337.5^\circ$

Find the exact value of each.

7) $\tan \theta = \frac{15}{8}$ where $180 \leq \theta < 270$

Find $\tan 2\theta$

8) $\cos \theta = -\frac{2\sqrt{14}}{9}$ where $90 \leq \theta < 180$

Find $\sin 2\theta$

9) $\tan \theta = \frac{\sqrt{2}}{4}$ where $0 \leq \theta < 90$

Find $\cos \frac{\theta}{2}$

10) $\tan \theta = \frac{\sqrt{3}}{3}$ where $0 \leq \theta < 90$

Find $\cos 2\theta$

11) $\cos \theta = \frac{3}{5}$ where $270 \leq \theta < 360$

Find $\cos 2\theta$

12) $\tan \theta = \frac{4}{3}$ where $180 \leq \theta < 270$

Find $\tan \frac{\theta}{2}$

Verify each identity.

$$13) \frac{\sin 2x}{\cos x} = 2\sin x$$

$$14) \frac{2\sin x \cos x}{\cos 2x} = \tan 2x$$

$$15) \frac{2\sin x \cos x}{\csc^2 x - 1} = \tan^2 x \sin 2x$$

$$16) \frac{\sin 2x}{\cot^2 x (1 + \cos 2x)} = \tan^3 x$$

Multiple-Angle Identities

Use the half-angle identities to find the exact value of each.

1) $\sin 292.5^\circ$

$$-\frac{\sqrt{2 + \sqrt{2}}}{2}$$

2) $\sin 337.5^\circ$

$$-\frac{\sqrt{2 - \sqrt{2}}}{2}$$

3) $\tan 105^\circ$

$$-2 - \sqrt{3}$$

4) $\cos 105^\circ$

$$-\frac{\sqrt{2 - \sqrt{3}}}{2}$$

5) $\cos 157.5^\circ$

$$-\frac{\sqrt{2 + \sqrt{2}}}{2}$$

6) $\cos 337.5^\circ$

$$\frac{\sqrt{2 + \sqrt{2}}}{2}$$

Find the exact value of each.

7) $\tan \theta = \frac{15}{8}$ where $180 \leq \theta < 270$

Find $\tan 2\theta$

$$-\frac{240}{161}$$

8) $\cos \theta = -\frac{2\sqrt{14}}{9}$ where $90 \leq \theta < 180$

Find $\sin 2\theta$

$$-\frac{20\sqrt{14}}{81}$$

9) $\tan \theta = \frac{\sqrt{2}}{4}$ where $0 \leq \theta < 90$

Find $\cos \frac{\theta}{2}$

$$\frac{\sqrt{18 + 12\sqrt{2}}}{6}$$

10) $\tan \theta = \frac{\sqrt{3}}{3}$ where $0 \leq \theta < 90$

Find $\cos 2\theta$

$$\frac{1}{2}$$

11) $\cos \theta = \frac{3}{5}$ where $270 \leq \theta < 360$

Find $\cos 2\theta$

$$-\frac{7}{25}$$

12) $\tan \theta = \frac{4}{3}$ where $180 \leq \theta < 270$

Find $\tan \frac{\theta}{2}$

$$-2$$

Verify each identity.

$$13) \frac{\sin 2x}{\cos x} = 2\sin x$$

$$\frac{\sin 2x}{\cos x} \quad \text{Use } \sin 2x = 2\sin x \cos x$$

$$\frac{2\sin x \cos x}{\cos x} \quad \text{Cancel common factors}$$

$$2\sin x \quad \blacksquare$$

$$14) \frac{2\sin x \cos x}{\cos 2x} = \tan 2x$$

$$\frac{2\sin x \cos x}{\cos 2x} \quad \text{Use } \sin 2x = 2\sin x \cos x$$

$$\frac{\sin 2x}{\cos 2x} \quad \text{Use } \tan 2x = \frac{\sin 2x}{\cos 2x}$$

$$\tan 2x \quad \blacksquare$$

$$15) \frac{2\sin x \cos x}{\csc^2 x - 1} = \tan^2 x \sin 2x$$

$$\frac{2\sin x \cos x}{\csc^2 x - 1} \quad \text{Use } \cot^2 x + 1 = \csc^2 x$$

$$\frac{2\sin x \cos x}{\cot^2 x} \quad \text{Use } \sin 2x = 2\sin x \cos x$$

$$\frac{\sin 2x}{\cot^2 x} \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\tan^2 x \sin 2x \quad \blacksquare$$

$$16) \frac{\sin 2x}{\cot^2 x(1 + \cos 2x)} = \tan^3 x$$

$$\frac{\sin 2x}{\cot^2 x(1 + \cos 2x)} \quad \text{Use } \sin 2x = 2\sin x \cos x$$

$$\frac{2\sin x \cos x}{\cot^2 x(1 + \cos 2x)} \quad \text{Use } \cos 2x = 2\cos^2 x - 1$$

$$\frac{2\sin x \cos x}{2\cot^2 x \cos^2 x} \quad \text{Cancel common factors}$$

$$\frac{\sin x}{\cot^2 x \cos x} \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\frac{\tan^2 x \sin x}{\cos x} \quad \text{Use } \tan x = \frac{\sin x}{\cos x}$$

$$\tan^3 x$$