

Mathematical Induction

Write the induction proof statements P_1 , P_k , and P_{k+1} for each conjecture.

1) $P_n: 11 + 19 + 27 + \cdots + 8n + 3 = n(4n + 7)$

2) $P_n: 7^n - 4^n$ is divisible by 3

3) $P_n: 4^n \geq 4n$

Use mathematical induction to prove that each statement is true for all positive integers

4) $36 + 324 + 900 + \cdots + (12n - 6)^2 = 12n(4n^2 - 1)$

5) 3 is a factor of $4^n + 2$

6) $7^n \geq 7n$

Mathematical Induction

Write the induction proof statements P_1 , P_k , and P_{k+1} for each conjecture.

1) $P_n: 11 + 19 + 27 + \dots + 8n + 3 = n(4n + 7)$

$$P_1: 8 + 3 = 4 + 7$$

$$P_k: 11 + 19 + 27 + \dots + 8k + 3 = k(4k + 7)$$

$$P_{k+1}: 11 + 19 + 27 + \dots + 8k + 3 + 8(k+1) + 3 = k(4k + 7) + 8(k+1) + 3$$

2) $P_n: 7^n - 4^n$ is divisible by 3

$$P_1: 7^1 - 4^1 \text{ is divisible by 3}$$

$$P_k: 7^k - 4^k \text{ is divisible by 3. Therefore, } 7^k - 4^k = 3r \text{ for some integer } r.$$

$$P_{k+1}: 7^{(k+1)} - 4^{(k+1)} \text{ is divisible by 3}$$

3) $P_n: 4^n \geq 4n$

$$P_1: 4^1 \geq 4 \cdot 1$$

$$P_k: 4^k \geq 4k \text{ for some positive integer } k \geq 1$$

$$P_{k+1}: 4^{k+1} \geq 4(k+1)$$

Use mathematical induction to prove that each statement is true for all positive integers

4) $36 + 324 + 900 + \dots + (12n - 6)^2 = 12n(4n^2 - 1)$

Let P_n be the statement $36 + 324 + 900 + \dots + (12n - 6)^2 = 12n(4n^2 - 1)$

Anchor Step

$$P_1 \text{ is true since } (12 - 6)^2 = 12(4 \cdot 1^2 - 1)$$

Inductive Hypothesis

Assume that P_k is true: $36 + 324 + 900 + \dots + (12k - 6)^2 = 12k(4k^2 - 1)$

Inductive Step

We now show that P_{k+1} is true:

$$\begin{aligned} 36 + 324 + 900 + \dots + (12k - 6)^2 + (12(k+1) - 6)^2 &= 12k(4k^2 - 1) + (12(k+1) - 6)^2 \\ &= 48k^3 - 12k + (12k - 6)^2 \\ &= 48k^3 - 12k + 144k^2 + 144k + 36 \\ &= 48k^3 + 144k^2 + 132k + 36 \\ &= 12(4k^3 + 12k^2 + 11k + 3) \\ &= 12(4k^3 + 12k^2 + 12k + 4 - k - 1) \\ &= 12(4(k^3 + 3k^2 + 3k + 1) - (k + 1)) \\ &= 12(4(k+1)^3 - (k+1)) \\ &= 12(k+1)(4(k+1)^2 - 1) \end{aligned}$$

Conclusion

By induction P_n is true for all $n \geq 1$.

5) 3 is a factor of $4^n + 2$

Let P_n be the statement 3 is a factor of $4^n + 2$

Anchor Step

P_1 is true: 3 is a factor of $4^1 + 2$

Inductive Hypothesis

Assume that 3 is a factor of $4^k + 2$. Therefore, $4^k + 2 = 3r$ for some integer r .

Inductive Step

We now show that P_{k+1} is true: 3 is a factor of $4^{(k+1)} + 2$

$$\begin{aligned} & 4 \cdot 4^k + 2 \\ & (3 + 1) \cdot 4^k + 2 \\ & 3 \cdot 4^k + 4^k + 2 \\ & 3 \cdot 4^k + 3r \\ & 3(4^k + r) \end{aligned}$$

Conclusion

By induction P_n is true for all $n \geq 1$.

6) $7^n \geq 7n$

Let P_n be the statement $7^n \geq 7n$

Anchor Step

P_1 is true: $7^1 \geq 7 \cdot 1$

Inductive Hypothesis

Assume that P_k is true: $7^k \geq 7k$ for some positive integer $k \geq 1$

Inductive Step

We now show that P_{k+1} is true:

$$\begin{aligned} & 7^k \geq 7k && k \geq 1 \\ & 7 \cdot 7^k \geq 7 \cdot 7k && 42k \geq 42 \geq 7 \\ & 7^{k+1} \geq 49k \\ & 7^{k+1} \geq 7k + 42k \\ & 7^{k+1} \geq 7k + 7 \\ & 7^{k+1} \geq 7(k + 1) \end{aligned}$$

Conclusion

By induction P_n is true for all $n \geq 1$.