

## Differentials

For each problem, find the differential  $dy$ .

1)  $y = -x^3 - 2$

2)  $y = -\frac{3}{x}$

For each problem, find the general formulas for  $dy$  and  $\Delta y$ .

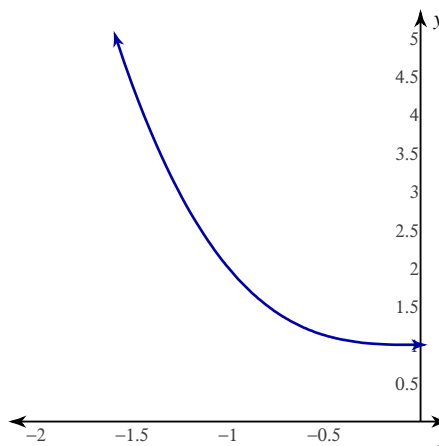
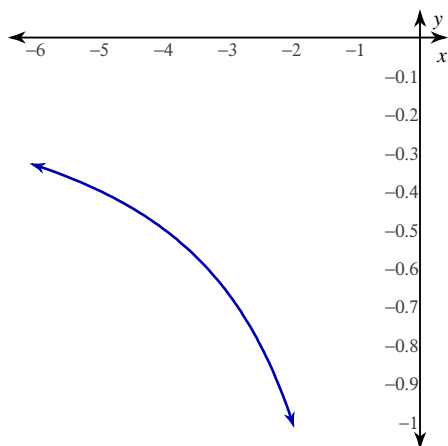
3)  $y = -x^3 - 2$

4)  $y = \frac{2}{x}$

For each problem, find  $dy$  and  $\Delta y$ , given  $x_0$  and  $dx = \Delta x$ . You may use the provided graph of the function to sketch  $dx$ ,  $\Delta x$ ,  $dy$ , and  $\Delta y$ .

5)  $y = \frac{2}{x}$ ;  $x_0 = -5$ ,  $dx = \Delta x = \frac{5}{2}$

6)  $y = -x^3 + 1$ ;  $x_0 = -1$ ,  $dx = \Delta x = -\frac{1}{2}$



**For each problem, find a linear approximation of the given quantity.**

7)  $\sin 122^\circ$

8)  $6.99^4$

**Use differentials to solve each problem.**

9) The radius of a sphere is measured to be 7 cm, with a possible error of  $\pm \frac{1}{10}$  cm. Estimate the possible propagated error in the calculated volume.

10) The sides of a square are measured to be 4 in, with a possible error of  $\pm \frac{1}{5}$  in. Estimate the possible propagated error in the calculated area.

## Differentials

For each problem, find the differential  $dy$ .

1)  $y = -x^3 - 2$

$$dy = -3x^2 dx$$

2)  $y = -\frac{3}{x}$

$$dy = \frac{3}{x^2} dx$$

For each problem, find the general formulas for  $dy$  and  $\Delta y$ .

3)  $y = -x^3 - 2$

$$dy = -3x^2 dx$$

$$\Delta y = -3x^2 \Delta x - 3x(\Delta x)^2 - (\Delta x)^3$$

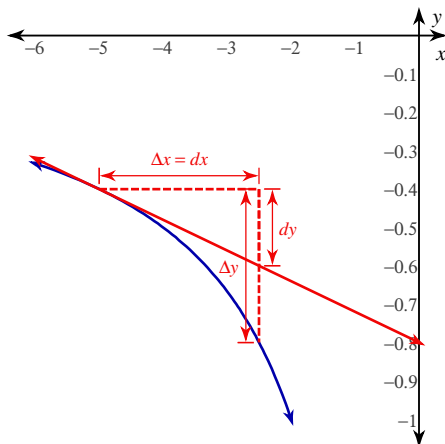
4)  $y = \frac{2}{x}$

$$dy = -\frac{2}{x^2} dx$$

$$\Delta y = -\frac{2\Delta x}{x^2 + x\Delta x}$$

For each problem, find  $dy$  and  $\Delta y$ , given  $x_0$  and  $dx = \Delta x$ . You may use the provided graph of the function to sketch  $dx$ ,  $\Delta x$ ,  $dy$ , and  $\Delta y$ .

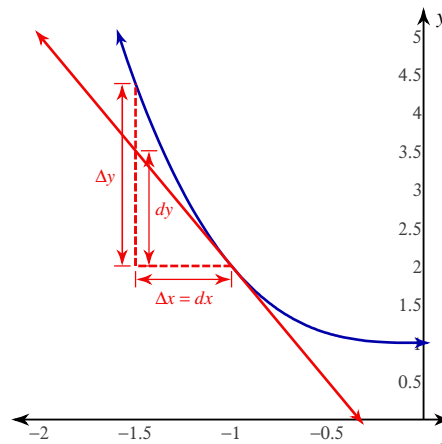
5)  $y = \frac{2}{x}$ ;  $x_0 = -5$ ,  $dx = \Delta x = \frac{5}{2}$



$$dy = -\frac{1}{5} = -0.2$$

$$\Delta y = -\frac{2}{5} = -0.4$$

6)  $y = -x^3 + 1$ ;  $x_0 = -1$ ,  $dx = \Delta x = -\frac{1}{2}$



$$dy = \frac{3}{2} = 1.5$$

$$\Delta y = \frac{19}{8} = 2.375$$

For each problem, find a linear approximation of the given quantity.

7)  $\sin 122^\circ$

$$f(x) = \sin x, f'(x) = \cos x$$

$$x_0 = \frac{2\pi}{3} \text{ radians}, \Delta x = \frac{\pi}{90} \text{ radians}$$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{90\sqrt{3} - \pi}{180} \approx 0.8486$$

8)  $6.99^4$

$$f(x) = x^4, f'(x) = 4x^3$$

$$x_0 = 7, \Delta x = -0.01$$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{59682}{25} = 2387.28$$

Use differentials to solve each problem.

- 9) The radius of a sphere is measured to be 7 cm, with a possible error of  $\pm \frac{1}{10}$  cm. Estimate the possible propagated error in the calculated volume.

$$V = \frac{4}{3}\pi r^3, dV = 4\pi r^2 dr$$

$$r = 7, dr = \pm 0.1$$

$$\Delta V \approx dV = \pm \frac{98\pi}{5} \approx \pm 61.5752 \text{ cm}^3$$

- 10) The sides of a square are measured to be 4 in, with a possible error of  $\pm \frac{1}{5}$  in. Estimate the possible propagated error in the calculated area.

$$A = s^2, dA = 2s ds$$

$$s = 4, ds = \pm 0.2$$

$$\Delta A \approx dA = \pm \frac{8}{5} = \pm 1.6 \text{ in}^2$$