Candle Center - Infinite Precalculus Name____________________

The Fundamental Theorem of Algebra

Date________________ Period____

State the number of complex zeros, the possible number of real and imaginary zeros, the possible number of positive and negative zeros, and the possible rational zeros for each function.

1) $f(x) = 5x^4 - 36x^2 - 81$

2) $f(x) = 15x^5 + 3x^4 + 140x^3 + 28x^2 + 45x + 9$

3) $f(x) = 5x^3 - x^2 - 5x + 1$

4) $f(x) = 3x^3 + 11x^2 + 5x - 3$

5) $f(x) = 10x^5 - 15x^4 + 12x^3 - 18x^2 + 2x - 3$

6) $f(x) = 5x^5 - 25x^4 + 46x^3 - 230x^2 + 9x - 45$

State the possible rational zeros and an interval in which all real zeros lie for each function. Then factor each to linear and irreducible quadratic factors.

7) $f(x) = 2x^4 - 11x^2 + 9$

8) $f(x) = 27x^3 + 1$

9) $f(x) = 2x^5 + 10x^4 - 13x^3 - 65x^2 - 7x - 35$

10) $f(x) = 5x^5 - 25x^4 - 26x^3 + 130x^2 + 5x - 25$

11) $f(x) = 5x^4 + 31x^2 + 6$

12) $f(x) = 2x^4 + 9x^2 + 7$

Find all zeros.

13) $f(x) = 2x^4 - 19x^2 + 24$

14) $f(x) = x^3 + 8$

15) $f(x) = 3x^5 - 6x^4 + 14x^3 - 28x^2 - 5x + 10$

16) $f(x) = 5x^4 + 6x^2 + 1$

17) $f(x) = 9x^5 - 15x^4 + 57x^3 - 95x^2 - 42x + 70$

18) $f(x) = 5x^4 - 7x^2 + 2$

Factor each to linear factors. One zero has been given.

19) $f(x) = 5x^5 + 49x^4 + 125x^3 + 113x^2 + 22x - 10;\ -4 + \sqrt{6}$

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The Fundamental Theorem of Algebra

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State the number of complex zeros, the possible number of real and imaginary zeros, the possible number of positive and negative zeros, and the possible rational zeros for each function.

1) $f(x) = 5x^4 - 36x^2 - 81$

# of complex zeros: 4
Possible # of real zeros: 4, 2, or 0
Possible # of imaginary zeros: 4, 2, or 0
Possible # positive real zeros: 1
Possible # negative real zeros: 1
Possible rational zeros:
$\pm 1, \pm 3, \pm 9, \pm 27, \pm 81, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5}, \pm \frac{27}{5}, \pm \frac{81}{5}$

2) $f(x) = 15x^5 + 3x^4 + 140x^3 + 28x^2 + 45x + 9$

# of complex zeros: 5
Possible # of real zeros: 5, 3, or 1
Possible # of imaginary zeros: 4, 2, or 0
Possible # positive real zeros: 0
Possible # negative real zeros: 5, 3, or 1
Possible rational zeros:
$\pm 1, \pm 3, \pm 9, \pm \frac{1}{3}, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5}, \pm \frac{1}{15}$

3) $f(x) = 5x^3 - x^2 - 5x + 1$

# of complex zeros: 3
Possible # of real zeros: 3 or 1
Possible # of imaginary zeros: 2 or 0
Possible # positive real zeros: 2 or 0
Possible # negative real zeros: 1
Possible rational zeros: $\pm 1, \pm \frac{1}{5}$

4) $f(x) = 3x^3 + 11x^2 + 5x - 3$

# of complex zeros: 3
Possible # of real zeros: 3 or 1
Possible # of imaginary zeros: 2 or 0
Possible # positive real zeros: 1
Possible # negative real zeros: 2 or 0
Possible rational zeros: $\pm 1, \pm 3, \pm \frac{1}{3}$

5) $f(x) = 10x^5 - 15x^4 + 12x^3 - 18x^2 + 2x - 3$

# of complex zeros: 5
Possible # of real zeros: 5, 3, or 1
Possible # of imaginary zeros: 4, 2, or 0
Possible # positive real zeros: 5, 3, or 1
Possible # negative real zeros: 0
Possible rational zeros:
$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{1}{10}, \pm \frac{3}{10}$

6) $f(x) = 5x^5 - 25x^4 + 46x^3 - 230x^2 + 9x - 45$

# of complex zeros: 5
Possible # of real zeros: 5, 3, or 1
Possible # of imaginary zeros: 4, 2, or 0
Possible # positive real zeros: 5, 3, or 1
Possible # negative real zeros: 0
Possible rational zeros:
$\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5}$

State the possible rational zeros and an interval in which all real zeros lie for each function. Then factor each to linear and irreducible quadratic factors.

7) $f(x) = 2x^4 - 11x^2 + 9$

Possible rational zeros:
$\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$
Real zeros lie in: $[-3, 3]$
Factors to: $f(x) = (2x^2 - 9)(x - 1)(x + 1)$

8) $f(x) = 27x^3 + 1$

Possible rational zeros: $\pm 1, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm \frac{1}{27}$
Real zeros lie in: $[-1, 0]$
Factors to: $f(x) = (3x + 1)(9x^2 - 3x + 1)$

9) $f(x) = 2x^5 + 10x^4 - 13x^3 - 65x^2 - 7x - 35$

Possible rational zeros:
$\pm 1, \pm 5, \pm 7, \pm 35, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{7}{2}, \pm \frac{35}{2}$
Real zeros lie in: [–7, 3]
Factors to: $f(x) = (x + 5)(x^2 - 7)(2x^2 + 1)$

10) $f(x) = 5x^5 - 25x^4 - 26x^3 + 130x^2 + 5x - 25$

Possible rational zeros: $\pm 1, \pm 5, \pm 25, \pm \frac{1}{5}$
Real zeros lie in: [–3, 6]
Factors to: $f(x) = (x - 5)(5x^2 - 1)(x^2 - 5)$

11) $f(x) = 5x^4 + 31x^2 + 6$

Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{6}{5}$
Real zeros lie in: [0, 0]
Factors to: $f(x) = (5x^2 + 1)(x^2 + 6)$

12) $f(x) = 2x^4 + 9x^2 + 7$

Possible rational zeros: $\pm 1, \pm 7, \pm \frac{1}{2}, \pm \frac{7}{2}$
Real zeros lie in: [0, 0]
Factors to: $f(x) = (x^2 + 1)(2x^2 + 7)$

Find all zeros.

13) $f(x) = 2x^4 - 19x^2 + 24$

$\left\{2\sqrt{2}, -2\sqrt{2}, \frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}\right\}$

14) $f(x) = x^3 + 8$

$\{-2, 1 + i\sqrt{3}, 1 - i\sqrt{3}\}$

15) $f(x) = 3x^5 - 6x^4 + 14x^3 - 28x^2 - 5x + 10$

$\left\{2, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, i\sqrt{5}, -i\sqrt{5}\right\}$

16) $f(x) = 5x^4 + 6x^2 + 1$

$\left\{i, -i, \frac{i\sqrt{5}}{5}, -\frac{i\sqrt{5}}{5}\right\}$

17) $f(x) = 9x^5 - 15x^4 + 57x^3 - 95x^2 - 42x + 70$

$\left\{\frac{5}{3}, i\sqrt{7}, -i\sqrt{7}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}\right\}$

18) $f(x) = 5x^4 - 7x^2 + 2$

$\left\{1, -1, \frac{\sqrt{10}}{5}, -\frac{\sqrt{10}}{5}\right\}$

Factor each to linear factors. One zero has been given.

19) $f(x) = 5x^5 + 49x^4 + 125x^3 + 113x^2 + 22x - 10;\ -4 + \sqrt{6}$

$f(x) = (x + 1)^2(5x - 1)(x + 4 - \sqrt{6})(x + 4 + \sqrt{6})$