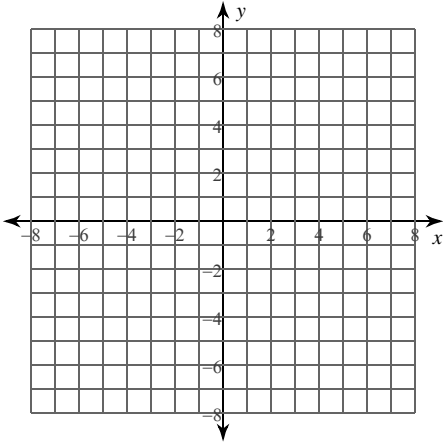


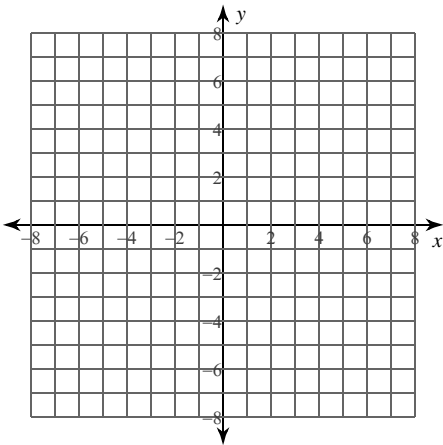
## Graphs of Rational Functions

For each function, identify the points of discontinuity, holes, intercepts, horizontal asymptote, domain, limit behavior at all vertical asymptotes, and end behavior asymptote. Then sketch the graph.

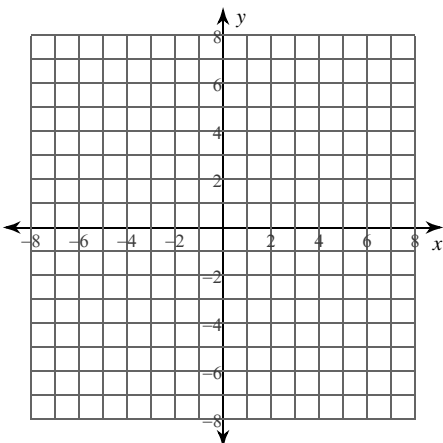
$$1) f(x) = \frac{1}{x-3} + 3$$



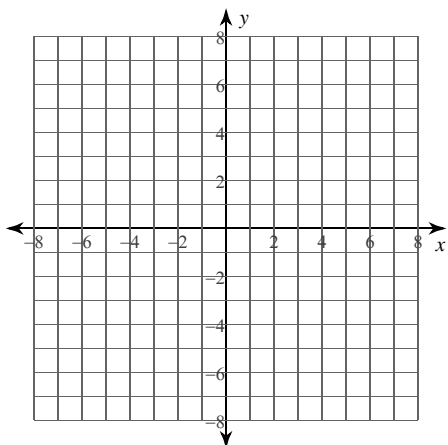
$$2) f(x) = -\frac{3}{x-2} - 2$$



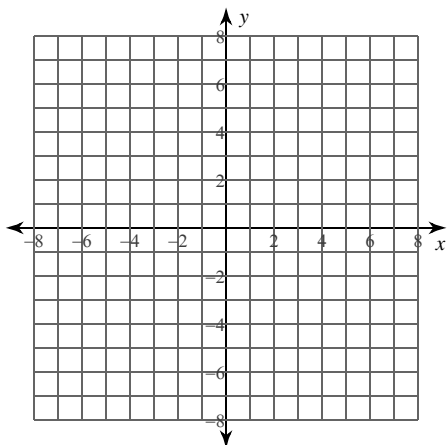
$$3) f(x) = \frac{x^2 - 4}{x^2 - 9}$$



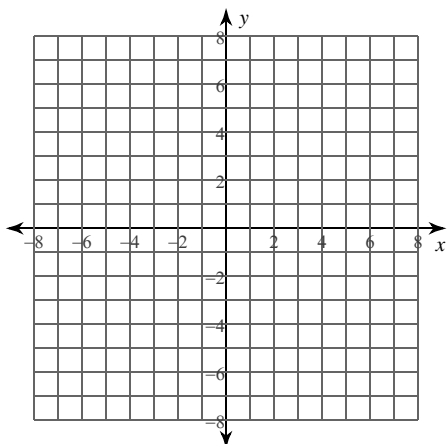
$$4) f(x) = \frac{2x^2 - 12x + 16}{x^2 - x - 12}$$



$$5) f(x) = \frac{x^2 + 2x - 3}{-3x - 6}$$



$$6) f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 8}$$

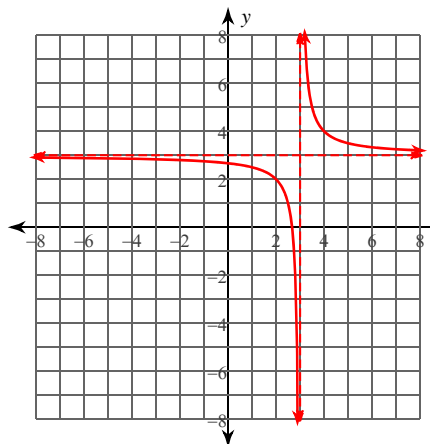


## Graphs of Rational Functions

Date \_\_\_\_\_ Period \_\_\_\_\_

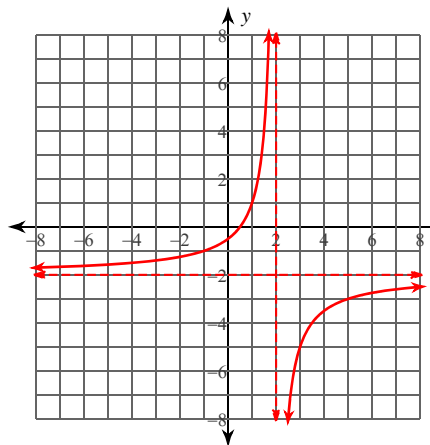
For each function, identify the points of discontinuity, holes, intercepts, horizontal asymptote, domain, limit behavior at all vertical asymptotes, and end behavior asymptote. Then sketch the graph.

$$1) f(x) = \frac{1}{x-3} + 3$$



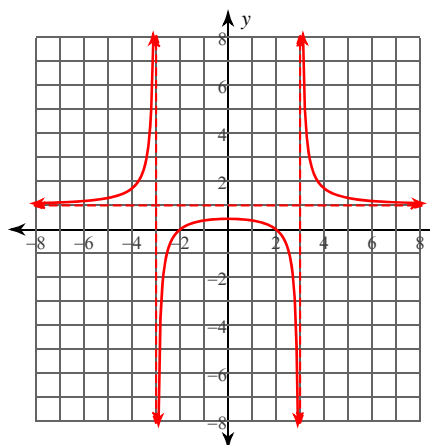
Discontinuities: 3  
 Holes: None  
 Horz. Asym.:  $y = 3$   
 x-intercepts:  $\frac{8}{3}$ , y-intercept:  $\frac{8}{3}$   
 Domain:  
 All reals except 3  
 Vert. Asym. behavior:  
 $\lim_{x \rightarrow 3^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow 3^+} f(x) = \infty$   
 End behavior asym.:  $y = 3$

$$2) f(x) = -\frac{3}{x-2} - 2$$



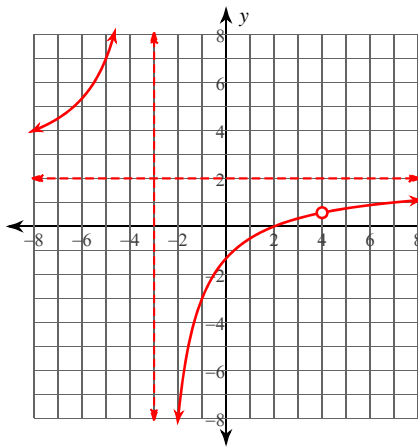
Discontinuities: 2  
 Holes: None  
 Horz. Asym.:  $y = -2$   
 x-intercepts:  $\frac{1}{2}$ , y-intercept:  $-\frac{1}{2}$   
 Domain:  
 All reals except 2  
 Vert. Asym. behavior:  
 $\lim_{x \rightarrow 2^-} f(x) = \infty$ ,  $\lim_{x \rightarrow 2^+} f(x) = -\infty$   
 End behavior asym.:  $y = -2$

$$3) f(x) = \frac{x^2 - 4}{x^2 - 9}$$



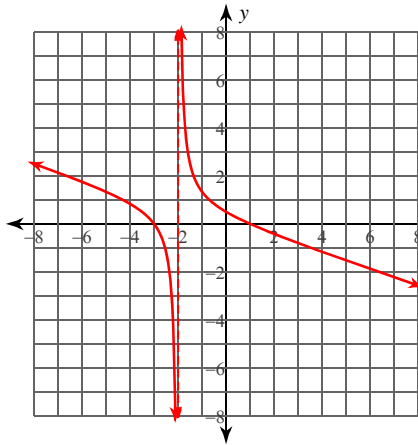
Discontinuities: 3, -3  
 Holes: None  
 Horz. Asym.:  $y = 1$   
 x-intercepts: 2, -2, y-intercept:  $\frac{4}{9}$   
 Domain:  
 All reals except -3, 3  
 Vert. Asym. behavior:  
 $\lim_{x \rightarrow -3^-} f(x) = \infty$ ,  $\lim_{x \rightarrow -3^+} f(x) = -\infty$   
 $\lim_{x \rightarrow 3^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow 3^+} f(x) = \infty$   
 End behavior asym.:  $y = 1$

$$4) f(x) = \frac{2x^2 - 12x + 16}{x^2 - x - 12}$$



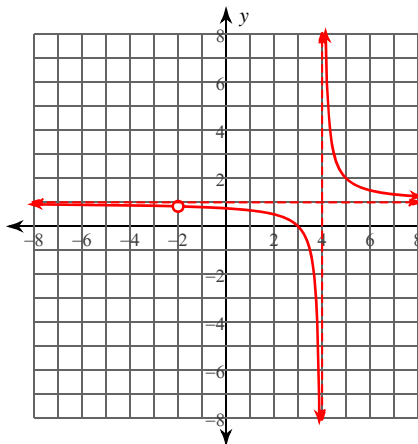
Discontinuities:  $-3, 4$   
 Holes:  $x = 4$   
 Horz. Asym.:  $y = 2$   
 x-intercepts:  $2$ , y-intercept:  $-\frac{4}{3}$   
 Domain:  
 All reals except  $-3, 4$   
 Vert. Asym. behavior:  
 $\lim_{x \rightarrow -3^-} f(x) = \infty$ ,  $\lim_{x \rightarrow -3^+} f(x) = -\infty$   
 $\lim_{x \rightarrow 4^-} f(x) = \infty$ ,  $\lim_{x \rightarrow 4^+} f(x) = -\infty$   
 End behavior asym.:  $y = 2$

$$5) f(x) = \frac{x^2 + 2x - 3}{-3x - 6}$$



Discontinuities:  $-2$   
 Holes: None  
 Horz. Asym.: None  
 x-intercepts:  $1, -3$ , y-intercept:  $\frac{1}{2}$   
 Domain:  
 All reals except  $-2$   
 Vert. Asym. behavior:  
 $\lim_{x \rightarrow -2^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow -2^+} f(x) = \infty$   
 End behavior asym.:  $y = -\frac{x}{3}$

$$6) f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 8}$$



Discontinuities:  $4, -2$   
 Holes:  $x = -2$   
 Horz. Asym.:  $y = 1$   
 x-intercepts:  $3$ , y-intercept:  $\frac{3}{4}$   
 Domain:  
 All reals except  $4, -2$   
 Vert. Asym. behavior:  
 $\lim_{x \rightarrow -2^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow -2^+} f(x) = \infty$   
 $\lim_{x \rightarrow 4^-} f(x) = \infty$ ,  $\lim_{x \rightarrow 4^+} f(x) = -\infty$   
 End behavior asym.:  $y = 1$